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OF AN ELECTION SYSTEM

By Alfred De Grazia

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BY ALFRED DE GRAZIA *

ON 16 June 1770, a French mathematician and navigator, Jean-Charles de Borda, who was later to serve the National Assembly on the commission that founded the metric system, turned his attention to social science in a *Mémoire sur les Élections au Scrutin*.[1] Taking as his stated purpose the design of a pure and just majority system, he presented algebraically two ways of achieving this goal. He then demonstrated how it would be possible beyond cavil to say that a candidate in an ordinary election by plurality had achieved a true majority whether the number of candidates were 2, 3, 4, or more. The paper, which was first printed in 1781, is presented below in translation from the French original. At the time of its delivery, it aroused considerable interest, and the election of officers of the Royal Academy of Sciences, before a session of which it was presented, was conducted for a time according to its principles.[2]

The present significance of Borda's memoir lies in its incorporation of key problems in the history of ideologies, in the theory of representative government, and in the logic of social science. Its specificity allows a remarkable view of the interrelatedness of philosophy and science, and encourages inductive speculations that ordinarily must be made from separate, dissimilar, and diffuse intellectual productions.

The French Enlightenment produced a great variety of pre-scientific and scientific

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[1] *Mémoires de l'Académie Royale des Sciences*, 1781, 657–65.

[2] States a commentator on the memoir, *Ibid.*, p. 34, "Les observations de M. de Borda, sur les inconvéniens de la méthode d'élire, presque généralement adoptée, sont très importantes & absolument nouvelles." Jean Mascart credited Borda for the method adopted soon thereafter by the Academy of Sciences. "La vie et les travaux du Chevalier Jean-Charles de Borda," *Annales de l'Université de Lyons*, Nouvelle Series, *2* (1919), fas. 33. He writes (p. 130): "Le procédé qu'il propose était, en fait, absolument nouveau pour l'époque et suscita une vive curiosité."

researches. Men thirsted for knowledge of all sorts, ranging from celestial mechanics to biology and including, in the area of human affairs, daring speculations on the nature of man and society. Those speculations are the inheritance of present-day social philosophy. One thinks immediately of Montesquieu and Rousseau, of rationalists, encyclopedists, democrats, and conservatives — of many men whose moral doctrines live today. But men of the Enlightenment are considered generally to have little relevance to what today is frequently called the "New Social Science."

The "New Social Science" is modelled in method upon the asserted objectivity and value-neutrality of natural science, and is aimed at the analysis of objective data with quantitative precision. If it is ascribed at all to pre-twentieth century influences, it is supposed to have originated in physiological psychology, statistical economics, and positivist sociology. Temporarily, at least, we are left with suppositions. Historians of science have neglected the dim trails by which the "New Social Science" approached the present. Yet it is plausible to conjecture that the Enlightenment, so prolific of scientific curiosity, would *have had* to try its hand at something resembling modern social science.

The case of the invention of an election system, which forms the subject of this study, would support the hypothesis. Indeed the case seems not to have been isolated or unexpected at the time. At least several brilliant scientists of the Enlightenment did venture confidently into the realm of society, prepared to reduce a complex reality to a precise and abstract order. Condorcet,[3] Laplace,[4] and others of unknown number and influence sought strenuously "connaissances 'pratiques' et 'usageables,' " as Daniel Mornet puts it,[5] of human behavior as well as of natural events.

Perhaps we now ignore this facet of their work because the new social science did not immediately emerge. Or perhaps we ignore it because we have little idea even now of what the social sciences are, and what we cannot define, we cannot trace. In either event, the case at hand, if at all representative, can teach us something of what social science meant to the Enlightenment. More than that, it shows that social science theory, which present standards would consider advanced, was even then being constructed. Finally, we may suspect that the difficulties of such theory are scarcely strangers to contemporary studies in social science.[6]

MEMOIR ON ELECTIONS BY BALLOT

BY JEAN-CHARLES DE BORDA (1733–1799)

It is an opinion generally held, and I know not whether it has ever been objected to, that in an election by ballot the plurality of voices indicates the will of the electors, that is to say, that the candidate who obtains such plurality is necessarily he whom the electors prefer to his competitors. But I am going to make it plain that this opinion, which is true in the case where the election is conducted between two candidates only, may lead to error in all other cases.

Let us suppose, for example, that an election be held among three subjects, A, B and C, and that the voters number 21. Suppose in addition that of the 21 electors, there are 13 who prefer B to A, and 8 who prefer A to B; that the same 13 electors also prefer C to A while the other 8 insist upon A over C; it is clear that A will have, in the collective opinion of the voters a marked inferiority, whether it be in reference to B or to C, since each of these latter, compared with A, has 13 votes while A has only 8; whence it would appear that the will of the electors would exclude A.

Nevertheless it will occur that in conducting the election in the ordinary manner, this candidate might have a plurality of votes. In effect, one cannot help but suppose that of the 13

[3] *Oeuvres complètes* (Paris, 1804), *8*, 194 ff.; *9*, 304 ff.; *12*, 427 ff.; *13*, 239 ff.; *8*, papers beginning on pp. 51, 240, 285, and 363. Condorcet was elected to the Academy of Sciences in 1769 and knew Borda's work well.

[4] *Théorie analytique des probabilités*, First Supplement, in *Oeuvres complètes* (Paris, 1878–1912), Vol. 7. Laplace served on the commission for standards of measure with Borda. I cannot say whether a "social science clique" existed nor guess who influenced whom.

[5] *Les origines intellectuelles de la révolution française*, Paris: Librairie Armand Colin, 1933, p. 474.

[6] The scholar who may wish to compare Borda's paper with similar recent works may begin by consulting: Duncan Black, "On the Rationale of Group Decision-Making," *J. Pol. Econ.*, *56* (1948), 23–24; L. S. Penrose, "Elementary Statistics of Majority Voting," *J. Royal Statis. Soc.*, No. I (1946), 53–7; John von Neumann and Oscar Morgenstern, *Theory of Games and Economic Behavior* (Princeton: Princeton U. Press, 1947); Kenneth J. Arrow, *Social Choice and Individual Values* (New York: Wiley, 1951); and see below, fn. 10.

electors who favor B and C and who give one or the other their preference over A, 7 may put B above C and 6 put C above B, in which event, totalling the votes, one would have the following result:

8 votes for A
7 " " B
6 " " C

Thus A would have a plurality, though, hypothetically, the opinion of the voters was against him.

In thinking about the stated example, one perceives that A has an advantage in the election results since B and C, who are his superiors, have divided almost equally the choice of 13 electors. One may compare them exactly to two athletes, who, after having exhausted themselves against each other, are subsequently vanquished by a third who is weaker than either of them.

The conclusion at which we arrive is that the ordinary method of conducting elections is seriously defective, and the defect arises from the fact that in this form of election the electors cannot make known in a sufficiently complete manner their opinion on the various candidates before them. In effect, when amongst numerous candidates, A, B, C, D, etc., one of the voters gives his vote to B, and another his vote to C, the first pronounces only on the superiority of B relative to his competitors and says nothing of the place he assigns to C among the others whom he does not name. Similarly the second, who accords to C his choice over all, goes no farther in declaring what place he gives to B; this cannot be regarded indifferently, because he of the two who obtains a more distinguished place among those unnamed, has, all things being equal, a reason for being preferred over the other, and in general the claim of each candidate to the choice of the electorate is the resultant of the different places which he occupies in the opinion of each elector; whence one perceives that for a form of election to be good, it must give to the voters a means of pronouncing on the merit of each candidate compared in succession to the merits of each of his competitors. Now, there are for that two forms of election which one may equally adopt; in the first, each elector assigns places to the candidates according to the degree of merit which he grants each of them; in the second, one conducts as many individual elections as there are combinations of the candidates taken two by two, and in that way may compare successively each candidate with all the others. It is easy to see that this latter form derives necessarily from the first, and that both reveal, as completely as possible, the opinion of the electors on all the candidates; but one must need know how to obtain the result of the vote in the two types of election; and it is this that I am going to examine in the rest of this memoir.

I will begin with the first type of election which I will call election by order of merit. Let us suppose first that there are only three candidates, and that each elector has written their three names on a ballot, arranging them following the degree of merit which he attributes to each, and that they mark the ballots

A A B C
B C A B
C B C A etc.

Consider first one of these ballots, for example, the first in which a voter has given the first place to A, the second to B, and the third to C, and I say that the degree of superiority which this elector has accorded A over B, may be counted the same as the degree of superiority which he accords B over C; since the second candidate B has received all the degrees of merit contained within the merits of the two other candidates A and C, one has no reason for stating that the elector who has ranked the three candidates, has wished to place him more or less near to A than to C, or, that which is the same thing, that he has attributed more superiority to the first over the second than he has given to the second over the third. Furthermore, because of the presumed equality of all the electors, each place assigned by one of the voters ought to be counted with the same value, and to achieve the same degree of merit as the place correspondingly assigned to another subject, or to the same by some other elector.

It follows from this that if one wants to represent by (a) the merit which each elector ascribes to the last place and by (a + b) that which he ascribes to the second, the merit which goes to the first must be represented as (a + 2b), and the same will do for the positions given by the other electors, of whom each last choice will equally be represented by (a), each second choice by (a + b) and each first by (a + 2b).

Let us suppose now that there are four candidates. One may demonstrate by the same reasoning, that the superiority of the first place over the second, that of the second over the third and that of the third over the fourth must be graded equal; and that the places correspondingly given by the various voters, suppose the same degree of merit; whence one concludes that the merits attributed by the electors to the fourth, third, second and first places can be represented by a, a + b, a + 2b, and a + 3b. It will be the same for a greater number of candidates.

Given the foregoing, it will be easy in any election to compare the value of the suffrages accorded the different candidates. To do so, one multiplies by (a) the number of last choices given to each candidate; by (a + b) the number of next-to-the-last choices; by (a + 2b) the number of the next preceding choice and so on; one may order all the different products for each candidate and the sum of these products will represent the value of the suffrages accorded him.

It is easy to see that in the question under discussion the quantities (a) and (b) can be anything one wishes and one may then suppose a to equal 1 and b to equal 1 and thus the value of the suffrages of each candidate will be represented by multiplying the number of last choices by 1, that of the next-to-the-last by 2, that of the next preceding choice by 3, and so on up to the number of first choice, which will be multiplied by the number of candidates.

Let us give an example of an election of this type; let us suppose again 21 electors and three candidates — A, B, and C, and the ballots emerge thus:

A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	C	C	C	C	C	C
B	C	C	C	C	C	C	C	C	C	C	C	C	C	C	B	B	B	B	B	B
C	B	B	B	B	B	B	B	A	A	A	A	A	A	A	A	A	A	A	A	A

One will obtain, as we have said, the comparative value of the suffrages by multiplying the first choice by 3, the second by 2 and the third by 1, which will give the following results:

Suffrages of A	8 first choices, multiplied by 3 = 24		37
	13 third " " " 1 = 13		
Suffrages of B	7 first " " " 3 = 21		42
	7 second " " " 2 = 14		
	7 third " " " 1 = 7		
Suffrages of C	6 first " " " 3 = 18		47
	14 second " " " 2 = 28		
	1 third " " " 1 = 1		

Whence one sees that the superiority of votes is in favor of C, that second place goes to B, and last place to A.

It may be remarked that if one were to conduct the election in the ordinary manner, one would have the following result:

8 votes for A
7 votes for B
6 votes for C

That is to say that the plurality would have been for A, who is last in the esteem of the voters, and that C, who is really first, would have had fewer votes than each of the others.

Let us suppose now that one wishes to use the method of individual elections, and that the three candidates, A, B, and C are given; as one can combine the three candidates two by two in three different ways, three individual elections are necessary. The results of these elections would emerge as follows:

First election between A and B	a	votes	for	A
	b	"	"	B
Second " " A and C	a′	"	"	A
	c′	"	"	C
Third " " B and C	b′	"	"	B
	c′	"	"	C

It is necessary now to find the comparative value of the votes accorded the three candidates. To do so, we will suppose that these elections are the outcome of an election by order of merit, which is always possible, since knowing the rank which each candidate occupies in the opinion of each elector, one can always determine the number of votes which he must have in an election conducted between him and some other candidate. Accordingly, let (y) be the number of first choices which candidate A will have had in this election by order of merit; (x) the number of second choices; and (z) the number of third choices. It is clear, therefore, that the value of the suffrages of A may be represented by $3y + 2x + z$; but $y + x + z$ = the total number of electors; let this number therefore = E, and one will have by eliminating (z), the value of the suffrages of A represented by $2y + x + E$ or simply by $2y + x$ since E is common to all three suffrages. Now I may remark that for each first choice that A may have obtained in the election by order of merit, he must have two votes in the individual elections; to wit, one in the election between A and B, and another in the election between A and C; that for each second choice he would have had in the election by order of merit, he would only have one in the individual elections; and for the third choices he would get nothing. Whence one concludes that the number of votes he will have in all the individual elections, to wit, $(a + a')$ will be equal to $(2y + x)$; but we begin to see that this quantity $(2y + x)$ represents the value of the suffrages in the election by order of merit; then the quantity $(a + a')$ will represent that also in the individual elections, that is to say, that the value of suffrages accorded to a candidate will be represented by the sum of the votes which he will have received in all the individual elections which concern him; this may be applied obviously to elections conducted among a greater number of candidates.

If one determines the values of a, a′, b, b′, c, and c′, after the supposition that the individual elections are the result of the election by order of merit recounted above, one will find

a = 8 b = 13 c = 13
a′ = 8 b′ = 8 * c′ = 13

and in consequence, one will have:

The suffrages of A or a + a′ = 16
" " " B " b + b′ = 21 *
" " " C " c + c′ = 26

which gives the same relationship among the three suffrages which had been given by the first type of election.

For the rest, we will remark here that the second form of election of which we speak, might be inconvenient in practice, whenever a large number of candidates would occur, because the number of individual elections which would then be necessary would be too great. Thereupon one must prefer the form of election by order of merit which is much more expedient.

I will close this memoir with an examination of one particular question relative to the ordinary manner of holding elections. I have made it evident that in these elections, the plurality of votes is not always a certain indication of the desire of the voters; but this plurality can be so great that it may not be possible for the choice of the voters to fall upon anyone but the man who has obtained the plurality. To determine in what case that happens, let (M) be the number of candidates; (E) the number of electors; A the Candidate who has the plurality; B he who, after A, has the largest number of votes; finally (y) the choices of candidate A; and (z) those of candidate B.

Let us suppose, following an election held by order of merit among all the candidates, it is clear that now A will have a number of first choices equal to (y) and that B will have a number equal to (z). Now the worst that might happen to A would be that the electors who did not give him first place, would give him last, and that those who did not give first place to B, would give him all seconds. In this case, if the value of the first places is represented by (m), that of the seconds by (m–1) and that of the last by (1), the value of A's suffrages = my + E − y; and that of the suffrages of B = mz + (m–1) · (E–z); it will then be necessary if the election is to fall to A that one have:

$$my + E - y > (mz - 1) \cdot (E - z), \text{ or}$$
$$y > \frac{z + (m-2) \cdot E}{m - 1}.$$

Let $m = 2$, one will have $y > 3$; that is, in a case where the election is held between two candidates alone, the candidate who has the plurality of voices is legitimately elected; thus in this case, but in this alone, the ordinary form of election gives an exact result.

Let us suppose that candidate B has all the votes which A doesn't have; then one will have $z = E - y$; placing this value in the above formula, one will have $y > E \cdot \frac{m-1}{m}$.

If, in this last expression, one makes $m = 3$, one will have $y = 2/3E$; that is, when there are three candidates, to be sure of having the approval of the electors, one must have more than two-thirds of the votes.

One will likewise find, when there are four candidates, that y must be greater than 3/4 of E and so on.

In conclusion, if the number of candidates be equal to the number of electors or greater, the above formula $y > \frac{(m-2) \times E + 3}{m-1}$ becomes $y > E - 1$, that is to say that then the election cannot be rigorously decided save by unanimity, a most extraordinary result which might justify the practice followed by one nation to the North in the election of its kings.

It remains for me to observe, in concluding this memoir, that all that we have said on elections applies equally to the deliberations of private bodies or companies. These deliberations are in effect nothing other than types of elections among different proposals. They are therefore subject to the same rules.

The Memoir's Relation to Election Theory

With Borda's memoir now in mind, we can analyze more concretely its various meanings. Borda's systems of voting incorporated certain salient traits of his revolutionary period. Especially during those later years of the eighteenth century could one conceive of the vote as he did, and as we can today, as something that could be manipulated quantitatively in equations. It took centuries for votes to be recorded and for a majority principle to be established in parliamentary elections. It took additional centuries for votes to be regarded as personal, that is, separable from property in land, from honorary titles, and from property holdings. It took another effort, though now events were rushing into the great transformations of the Enlightenment, for a vote to be abstracted to the point of weighting choices.

Borda's treatment of the vote, then, as an abstractable and manipulable symbol was radical.[7] It was associated with the democratic temper of the French Revolution in

* In the original text, these figures appear, respectively, for 8, 13, and for 21, 12. I have changed them to what I believe would accord with de Borda's intentions.

[7] Unfortunately, one can only conjecture on the contents of a Spanish work which Señor Vallejo, Secretary of the Spanish Embassy, sent to the Academy of Sciences on 16 Vendemiaire

a way to delight historians who believe in the basic connections of quantitative science, rationalism, individualism, and democracy.[8] For the accompaniments of Borda's view of the vote are egalitarianism, mechanicalism, and the majority principle. Without assuming that one vote equals every other single vote, that is, one opinion every other opinion, he would hardly have moved into this field. Present wide acceptance of this belief should not blind us to its importance when it appeared in the late eighteenth century. This fiction not only subsumes the motivation and general character of the work, but also is used as a deliberate postulate. His postulation of equal intervals of choice as verifiable (and just!) carries him through a critical stage in the development of his scheme where he must weight numbered preferences of voters.[9]

The memoir bears yet another mark of the Enlightenment. It assumes briskly that a method of social action can be intellectually constructed. This assumption is reminiscent of the thinking of Rousseau, Diderot, Condorcet, Paine, Fénélon, d'Argenson, Holbach, and d'Alembert, but contrasts sharply with that of Montesquieu, Burke, and Taine. Whereas Borda and the former group regarded themselves as the architects of rational social institutions, the latter were impressed by the traditional and evolutionary nature of the social organism.

Borda's article, insofar as it has remained unknown to American political scientists, can contribute a crucial bit of evidence to the problem of tracing theories of representation from the medieval inalienable identification of the vote with the land, church and corporate titles, to the late 19th-century proliferation of systems of "proportional representation," based on completely abstracted votes, cast by casual and mobile electors who rank their preferences. Between the medieval French Estates General, revived briefly in 1789, and the recent Hare system of New York City or the Nanson Preferential Majority System,[10] Borda's mémoire stands as a sort of intellectual watershed. Although he intended to perfect the majority system, he gave an important practical invention to the critics of the majority system. This was the vote by numbered choices. So that, while the advocates of "PR" frequently (and wrongly, I believe) cite Mirabeau as their first inspiration for having said that the assembly ought to be a map-like miniature of the nation, in fact, a basic postulate for sophisticated systems of proportional representation, preferential voting, was the unique contribution of Borda.[11]

An. VI (Sept. 1797). It had the intriguing title, "Memoria matemática sobre el cálculo de la opinión en las elecciones." Upon its receipt, it was placed in the library of the Academy and Borda was asked to give a talk on it, but appears never to have done so. Mascart, *op. cit.*, pp. 130–1.

[8] Cassirer, *Myth of the State*, Chap. XIV; Mornet, *op. cit.*; Henri Eugène Sée, *L'évolution de la pensée politique en France au XVIII^e siècle* (Paris, 1925), pt. 5, chap. 21; J. S. Schapiro, *Condorcet and the Rise of Liberalism* (New York: Harcourt Brace, 1934); John Dewey, *The Public and Its Problems* (Chicago: Gateway Books, 1946), 85–109.

[9] Other postulates could be used for weighting differently the intervals among the first, second and succeeding choices, as Condorcet pointed out (*Oeuvres complètes*, *8*, 568). Given some empirical evidence on the distribution of intensities and the character of intensities among electors, a probability curve might have been employed as the basis for weighting the choices in a final tally. It should be noted as a defect of Borda's plan that as the number of candidates increased, the value of a first choice in relation to the next choice decreased, 2–1, 3–2, 4–3, 5–4, etc. The number of candidates would then become a matter of vital interest to the contending factions just as the advent of a third candidate into a two-sided plurality contest becomes so.

[10] Cf. Belle Zeller and Hugh A. Bone, "The Repeal of PR in New York City, "*Amer. Pol. Sci. Rev.*, *42* (1948), p. 1127 and J. B. Baldwin, "The Technique of the Nanson Preferential Majority System of Elections," 39 *Proc. Royal Society of Victoria* (1926, new series), 42–52.

[11] Cf. Simon Sterne, *On Representative Government and Personal Representation* (Philadelphia, 1871), 50–1; Mirabeau declared (*Oeuvres de Mirabeau*, Paris: Lecointe et Pougin, 1834–5; intro. by M. Merilhou, p. 7); "Les assemblées sont pour la nation ce qu'est une carte réduite pour son étendue physique: soit en petit, soit en grand, la copie doit toujours avoir les mêmes proportions que l'original." Quoting this and citing Esmein (*Droit Constitutionnel*, 5th ed., p. 272), Theodore Petitjean declares that the idea of proportional representation is not contained either in Article 6 of the Declaration of the Rights of Man or in Mirabeau's speech. For Mirabeau said in the same speech, "Lorsque la nation est trop nombreuse pour être réunie en une seule assemblée, elle en forme plusieurs, et tous les individus d'une même assemblée donnent à un seul le droit de voter pour eux." Comments Petitjean: "C'est bien là le système majoritaire, sans aucun correctif." (*La Représentation Proportionnelle devant les Chambres Françaises*, Librairie de la Société du

Preferential marking of ballots, a multi-member constituency, and an electoral quota are the three key elements of the rationalistic proportional representation systems of a later day. The latter two elements are to be found in Condorcet's writings, though never in a conscious relationship designed to insure proportional representation.[12] We can say that scarcely was the majority principle registering its triumph over men's minds when its opposing principle (minority representation) began to work its way. Indeed we are inclined to see in this fleeting episode confirmation of Hegel's "giant" theory that each human movement carries with it the seeds of its dissolution, inseparably attached to it.

More specifically in line with Borda's intention to implement the majority idea has been the development since his time, especially in England, the Dominions, and America of the single alternative vote.[13] In a common form, this procedure allows a voter to rank the candidates by preference. If no candidate receives a majority of first choices, the low candidate is dropped and his second choices parcelled out. If no majority is yet achieved, the process is repeated with the next lowest candidate, and so on, until a majority candidate is found and declared elected. The Borda election by order of merit leads rather easily into this sort of system.

Borda also developed for the first time, to my knowledge, the nucleus of the idea which later underlay the French system of "ballottage" and the American system of "run-off" elections, both of which attempt to encourage or insure a majority by successive elections.[14]

Weaknesses of Borda's Proposals

If the place of Borda's work in the history of science and representative government is of interest, so also are his failures at solving a problem of human relations by an exact and inflexible remedy. There are several weaknesses of an inter-related logical-ideological order that may be brought out and commented upon.

1. Borda postulates the "majority principle" but does not define it. Borda may or may not have its definition clearly in mind. If he does, then his postulate will only be

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Recueil Sirey, Paris, 1915, 234–5). A good general treatment of problems of representation in the constituent Assembly of 1789 is to be found in Karl Loewenstein, *Volk und Parlament nach der Staatstheorie der französischen Nationalversammlung von 1789* (Munich: Drei Masken Verlag, 1922). Nicholas Saripolos, *La démocratie et l'élection proportionelle*, Paris: Rousseau, 1899, writes that Rousseau may be called the father of proportional representation. "Rousseau par sa théorie sur l'état, le citoyen, l'égalité, peut être considéré, bien qu'adversaire de toute espèce de représentation, comme précurseur de la représentation *nationale*, *atomistique*, *proportionnelle*." Cf. pp. 159 ff., 526 ff., 773–4, 994. What he is saying here is substantially in agreement with our theory that the roots of theory to come are contained in the majority theory of Borda, which itself is a product and representative of its age.

One cannot resist adding here a quotation from John Adams, preceding (1776) the Mirabeau speech and quite similar to it: "Reason, justice, and equity never had weight enough on the face of the earth to govern the councils of men. It is interest alone that does it, and it is interest alone which can be trusted. . . . Therefore the interests within doors should be the mathematical representation of the interests without doors." Edmund Cody Burnett, *The Continental Congress* (New York: Macmillan, 1941), p. 224.

[12] E.g., *Oeuvres complètes*, *8*, 198. See Ernest Naville, "Les progrès de la représentation proportionnelle," *Le Représentation Proportionnelle*, 1883–5, for a series of articles summarizing the many steps to the theory of proportional representation in the nineteenth century. Condorcet's ideas were influential in the adoption by the Constitutional Convention in 1793 of the system of indirect elections. He not only knew of Borda's ideas but also knew and admired the American system of indirect elections for the election of the President.

[13] See H. F. Gosnell, *Democracy: Threshold of Freedom* (New York: Ronald Press, 1947), 169–70, 302.

[14] In his critique of Borda's ideas, Condorcet warns that if the number of electors is large and especially if the number of candidates is not very small, the method of successive elections is long and difficult to administer. (*Oeuvres complètes*, *8*, 573–4; *9*, 304–5.) He suggests limiting the number of candidates by nominating petitions, although he is antagonistic to parties or factions. The American run-off election is more ruthless in eliminating candidates, limiting the later election to the top two in the primary, while the French ballottage allowed those who wished to try again to do so, despite their not being among the top two. Karl Braunias, *Das Parlamentarische Wahlrecht* (Berlin: de Gruyter, 1932), *2*, chap. 1, describes the defects experienced under this system.

acceptable to those who agree with it.[15] It is *their* majority principle, and not an absolute and universally agreed to postulate of all engaged in the political process. Speaking historically and realistically, the majority principle is an ethical preference of considerable practical ambiguity.[16] It may be termed a practical fiction denoting the end result of many political motives.

Borda's usage of the term in his election by order of merit certainly prescribes a quick and ruthless mathematical transcription and tabulation of votes. Yet another preference for the majority principle (among the many known preferences) may demand a lengthy consultation of constituents and persuasion of minorities, with the tabulation of votes as an intermediate procedure and with a subsequent retabulation on the basis of a secondary election. This sort of preference is actually provided by Borda himself in his election by successive choices among pairs of candidates. Although he does say that his second system would be more cumbersome than his first plan, he ignores the many other factors that would make the "majority principle" effected by the first different from that produced by the second. In political campaigns, deals and combinations of many kinds would be possible if the second plan of successive elections were followed.

2. Borda reasons badly when he takes the preferential rating of 1,2,3, to represent equal intervals of intensity of preferences. He may say correctly (within the confines of his paper) that "one has no reason for stating that the elector who has ranked the three candidates, has wished to place him (B) more or less near to A than to C." But this negative statement does not justify its opposite, which is exactly what Borda goes on to assume, for he gives proportionate weights to first, second, third and all preferences and to that extent distorts the "mirror" of the electorate.

It must be admitted that this assumption is a quality of all voting systems that do not "force" consensus, including the simple plurality system.[17] Still, it may be rightly added that, while all equal voting systems measure social tension very crudely, those of Borda exaggerate crudeness by fixing it into a formal framework.

3. In another place, Borda points out that the traditional plurality vote would give A eight votes, B seven, and C six, while his own would allow every vote to give its appropriate weight to all candidates. Thus a refined inflexibility is substituted for rough inflexibility. The author overlooks the probability that new and different power situations would result. Nor perhaps was he aware of a common behavioral phenomenon of elections: many voters, when faced with more than two candidates, tend to vote for a less preferred one if their favorite seems certain to lose. The "Don't waste your vote" idea has often caused third party candidates in American elections to demonstrate electoral strength below their actual strength in adherents.[18]

4. Going back to the behavioral differences that would probably characterize the method of order of merit and the method of successive choices, we are prepared to challenge Borda's mathematical reasoning when, taking the voters' choices under the first election, he transfers them as equivalents to the formula he is developing to depict

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[15] H. F. Gosnell and this writer examined a number of question-begging definitions of representation in Chap. 8 of *Democracy: Threshold of Freedom*. Also cf. Alfred de Grazia, *Public and Republic* (New York: Knopf, 1951) and Lindsay Rogers, *The Pollsters* (New York: Knopf, 1949), first part, for prophylaxis in connection with the term "public opinion."

[16] See Braunias, *op. cit.*; J. G. Heinberg, "Theories of Majority Rule," *Amer. Pol. Sci. Rev.*, 26 (1932), 452–69; W. Kendall, *John Locke and the Doctrine of Majority Rule* (Urbana: Univ. of Illinois, 1941); and Alfred de Grazia, *Elements of Political Science* (New York: Knopf, 1952), 135–41.

[17] An entirely non-mechanical phenomenon that seems to adjust intensity of feeling to election results is non-participation. "Taking the trouble to vote" may correlate roughly with intensity of feeling in reference to the candidates. Cf. Angus Campbell and R. L. Kahn, *The People elect a President*, Survey Research Center Study, No. 9 (Ann Arbor: Univ. of Michigan Press, May, 1952).

[18] A comparison of pre-election polls and election results sometimes indicates this. However, other factors intervene, such as poll-place intimidation of third party voters and the failure of election officials of the major parties in America to tally third party votes.

the results of the second. We may then question his finding that both would give the same results. Most election systems that have been subsequently devised have committed this same error of transferring symbolic, legal, or formal procedures and presuming the equivalent transfer of the behavioral accompaniments of the initial procedures.

5. In another context, we can question Borda on his ambiguous use of postulates. Obviously one cannot object to a scientific postulate aimed at isolating a portion of the data. This technique can be judged by its observable scientific utility. An additional burden accrues, however, to a postulate which is made to perform two functions, a scientific one and a social one. In this case, the postulates of the majority principle and the equal intervals of choice, by leaving out many data that have a claim to relevance (which may be defined roughly as the interests of all actual attitudes towards the election process being discussed) may be mathematically-procedurally useful in proportion as they are social scientifically incomplete. So it is with Borda's plans: they forget that what science pulls asunder, practice must reconstruct.

6. Borda's two election forms, like a number of proposed systems since then, are essentially tautological. They define what was premised, either consciously or unconsciously. The election systems he describes are really operational definitions of his majority principles. When such election systems are put into practice, their character as intellectual tautologies is forgotten and their partial descriptions of behavior are thought to be sufficient to the whole problem originally attacked. The balance of behavior that subsequently associates itself with the now enacted system will in effect revise and reshape the scheme's *a priori* description. That is, the fullness of reality will transform the scheme. This is foreseen inadequately because of the ideological blinders worn by system inventors. A number of election devices, including various forms of proportional representation, have produced results in excess of or in contrast to those visualized before their enactment. For example, J. S. Mill, an astute advocate of proportional representation, believed independent voting would operate under it to the detriment of party voting In fact, "PR" has meant no decrease in party or group affiliation or responsibility, or in party or group control of candidates.

The term "election" covers a variety of situations, many of them highly complex forms of decision that have been long pre-histories, complicated electoral happenings, and significant post-electoral events. It is too much to hope now, as it was indeed in Borda's time, for a series of postulates that could encompass the possible varieties of behavior or for a statistical calculus of probabilities in the development of a theoretical election scheme governing the tendencies of observed behavior at all significant points where behavioral manifestations take place. Mathematical developments of election techniques have gone little beyond Borda, and, where they have been applied, their results have been scarcely better predicted. A few general behavioral forecasts can now be advanced with reference to new systems of voting such as the Hare system of proportional representation, the plurality system, and several varieties of list systems of proportional representation. But several crucial results of them are hotly disputed. Proportional representation, for example, which is the savior of democracy to some, is the "Trojan Horse" of democracy to others.

Finally, we may point out the enthusiasm of Borda and his ideological successors for "over-neutralizing" and "objectifying" political structures.[19] An election system is never neutral. It always discriminates against some interest and favors some other. The kind of "majority" that would succeed under Borda's first plan would differ from

[19] We ought to make clear that, by using the words "over-neutralizing" and "objectifying" we mean the ideological derivatives of the *Aufklärung*, not the essential natures of any and every *Weltanschauung*. The essence of a basic ideology, of course, is the fact that to its unconscious adherents, its view is real, i.e., objective. One of the finest statements of this occurs in José Ortega y Gasset, *Concord and Liberty* (New York: Norton, 1946; trans. by Helene Weyl), 170–1.

the kind of majority that would succeed under his second plan and from any other kind of majority-insuring scheme.

It is futile to try to de-politicize election machinery. The most "apolitical" machinery is that which is adjusted so well to the social equilibrium that it *seems* objective. Nor can a rigid electoral scheme lock the political process. John Dewey put it well when he declared that even professedly empirical philosophies have been non-historical. "They have isolated their subject-matter from its connections, and any isolated subject-matter becomes unqualified in the degree of its disconnections. In social theory dealing with human nature, a certain fixed and standardized 'individual' has been postulated, from whose assumed traits social phenomena could be deduced." [20] Objectifying and rationalizing electoral institutions does not transfix a political order. This was a common sociological fallacy among early democrats in England, France, and America. The piling of law upon law, of procedure upon procedure, of constitution upon constitution, is a repetitive culture complex, symptomatic of a larger and more meaningful culture. In Borda's time, the larger culture was a rising movement of science and liberal democracy.

In times of social change and stress, procedural fixation is so much chaff to the wind. When Bonaparte took office as President of the First Class of the Mechanical Section of the Institute (March, 1800), "il critiqua vivement en des termes qui ne nous sont pas connus" Borda's method of election.[21] After a long discussion, everyone agreed. A committee was appointed to study the matter, it recommended a new method, and Borda's system some time later marched into the dead files of history.

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[20] John Dewey, *The Public and Its Problems*, 194–5.

[21] Mascart, *op. cit.*, p. 133.